The Effect of Rotation on the Free Vibrations in a Non-homogeneous Orthotropic Elastic Hollow Sphere

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Abstract

This article examines the radial vibrations of spherical isotropy embedded in an elastic medium according to the one-dimensional (1D) elastic theory. Based on the linear theory of elasticity, the rotation and inhomogeneity effects on wave propagation in orthotropic material are analyzed. The 1Delastodynamicsequation is solved in terms of radial displacement. We consider three boundaries: free, fixed, and mixed orthotropic materials. In the case of harmonic vibrations, the eigenvalues of the natural frequency of the radial vibrations for different boundary conditions are determined. For each case, the numerical results are presented, illustrated graphically, and then compared with those in the absence of rotation and non-homogeneity. An increase in the rotation and non-homogeneity parameters is observed, similar to the findings of the classical sphere theory. Therefore, this study can also be used in the design and optimization of microplates and nanoplates. The findings show that rotation and non-homogeneity have a strong impact on wave propagation in orthotropic material.

Keywords: Free vibrations, Stresses, Rotating, Non-homogeneous, Orthotropic material, Wave propagation

1. Introduction

Due to flexural vibration, accidental failure of rotating sphere wheels hasfrequently occurred in rotodynamic machinery, such as steam and gas turbines. Hollow spheres are widely used as structural components because of their vibration characteristics that are crucialin practical design. A general theory of elasticity for a spherically isotropic medium with varying coefficients was introduced for purely elastic materials, which considered the free vibration problem of a nonhomogeneous spherically isotropic hollow sphere. Abd-Alla and Mahmoud ([1], [2]) examined the magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic heat conduction model and rotation effect on thermoelastic wave propagation in a nonhomogeneous infinite cylinder of isotropic material. Abd-Alla et al. ([3], [4]) analyzed wave propagation modeling in cylindrical human long wet bones with cavity and the rotation effect on the radial vibrations in a nonhomogeneous orthotropic hollow cylinder, whereas Mahmoud ([5], [6]) examined wave propagation in cylindrical poroelastic dry bones and the non-homogeneous effect on wave propagation in orthotropic elastic media. Mahmoud et al. [7] discussed the rotation effect on the radial vibrations in a nonhomogeneous orthotropic hollow cylinder, whereas Abd-Alla et al. [8] studied the rotation effect on a nonhomogeneous infinite cylinder of orthotropic material. The method has been widely used for static and free vibration analysis of beams [9]. Pradyumna and Bandyopadhyay [10] conducted free vibration analysis of functionally graded curved panels using a higher-order formulation.

Moreover, Wang et al. [11] studied the transient responses of homogeneous hollow spheres under radial deformation. Using the Ritz method and Chebyshev polynomials, Zhou et al. [12] investigated three-dimensional vibrations of solid and hollow cylinders and proposed a semi-analytical solution of free vibration of a finite cylinder. Mofakhami et al. [13] explored finite cylinder vibrations under different end boundary conditions and Ding et al. [14] evaluated the dynamic responses of a functionally graded poroelastic hollow sphere for spherically symmetric problems. Hollow spheres are frequently used in engineering industries; one of the fundamental problems in elasto-dynamics has been the corresponding free vibration problem. A more general model has recently been introduced, where it is assumed that Young's module and density of the orthotropic materials of the shells vary continuously and piecewise continuously in the thickness coordinate; this model has resolved the static and dynamic stability problems of single-layer and laminated orthotropic cylindrical and conical shells with simple or freely supported edges ([15], [16]).

Furthermore, Tang and Cheng [17] investigated an eigenfunction expansion method for the elastodynamic response of an elastic solid with mixed boundary surfaces. Chen and Ding [18] studied some coupled vibration problems of spherically isotropic hollow spheres. Kwak et al. [19] proposed a dynamic model of the cylindrical shell structure suitable for vibration suppression control. Chen et al. [20] examined free vibration of non-homogeneous transversely isotropic magnetoelectroelastic plates. Argatov [21] investigated the approximate solution of the axisymmetric contact problem for an elastic sphere. Huang and Ho [22] presented the first known analytical solution for vibrations of a polarly orthotropic Mindlin sectorial plate with simply supported radial edges, while Towfighi and Kundu [23] studied elastic wave propagation in anisotropic spherical curved plates. Zhang and Hasebe [24] investigated an elasticity solution for a radially nonhomogeneous hollow cylinder. Theotokoglou and Stampouloglou [25] analyzed the radially nonhomogeneous axisymmetric problems. Several hypotheses have been introduced and studied by many practitioners and researchers ([26]-[31]).

In this study, the 1Delastodynamicsequation for orthotropic media is solved in terms of displacement potentials. In addition, the eigenvalues of the natural frequency of the radial vibrations of the spherical body (solid-hollow) are determined for different boundary conditions in cases of harmonic vibrations. Numerical results of the frequency equation are represented and illustrated in detail for different cases (see figures). The findings prove that the rotation and in-homogeneity have a strong impact on natural frequency.

2.Formulation of the Problem

For a spherically orthotropic elastic medium, the spherical coordinates(r, θ, φ) are used, where r , θ, and φ are radial, co-latitudinal, meridional coordinates, respectively. The basic spherical orthotropic equationswere in agreement with the origin. We have only the radial displacement $u_r = u, u$as a function of r and $t$only,the circumferential displacement$u_\theta = 0$, and the longitudinal displacement $u_\varphi = 0,$ which are independent of θ and φ .

Stress components are as follows:

$$\sigma_{rr} = c_{11}\frac{\partial u}{\partial r} + c_{12}\frac{u}{r} + c_{13}\frac{u}{r},$$

$$\sigma_{\theta\theta} = c_{12}\frac{\partial u}{\partial r} + c_{22}\frac{u}{r} + c_{23}\frac{u}{r}, \tag{1}$$

$$\sigma_{\varphi\varphi} = c_{13}\frac{\partial u}{\partial r} + c_{23}\frac{u}{r} + c_{33}\frac{u}{r},$$

$$\tau_{r\varphi} = \tau_{r\theta} = \tau_{\theta\varphi} = 0.$$

We take the rotation terms about the z-axis as a body force; then, the dynamical equation in r direction is as follows:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\left(2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\varphi\varphi}\right) + \rho\Omega'^2 u = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2}$$

where ρ is the density of the material of the sphere and Ω' is the uniform angular velocity. We characterize the elastic constants $c_{ij}$and the density $\rho$of non-homogeneous material in the following:

$$c_{ij} = \alpha_{ij}r^{2m}, \qquad \rho = \rho_0 r^{2m}, i, j = 1,2,3, \tag{3}$$

where α_{ij} and ρ_0 are the values of $c_{ij}$and ρ in the homogeneous case, respectively, and

m is the non-homogeneous parameter.

Substituting (6) and (4) into (5), we obtain the following:

$$\frac{\partial^2 u}{\partial r^2} + \frac{2(m+1)}{r}\frac{\partial u}{\partial r} + \left(\frac{(2m+1)(\alpha_{12}+\alpha_{13}) - (\alpha_{22}+\alpha_{33}+2\alpha_{23})}{\alpha_{11}r^2} + \frac{\rho_0}{\alpha_{11}}\Omega'^2\right)u = \frac{\rho_0}{\alpha_{11}}\frac{\partial^2 u}{\partial t^2}. \tag{4}$$

In Section 3, the analytical solution for radial vibration of an elastic spherical body of orthotropic material is investigated.

3. Solution of the Problem

Here, the analytical solution of the above problem for a spherical region of inner radius a and outer radius b with different boundary conditions (free-fixed-mixed) is obtained by considering the harmonic vibrations. We assume the solution of (4) as follows:

$$u(r, t) = U(r)e^{-i\omega t}, \tag{5}$$

where ωis the natural frequency of the vibrations.Substituting (5) into (4), we obtain the following:

$$\frac{d^2U}{dr^2}+\frac{2(m+1)}{r}\frac{dU}{dr}+\left(\frac{(2m+1)(\alpha_{12}+\alpha_{13})-(\alpha_{22}+\alpha_{33}+2\alpha_{23})}{\alpha_{11}r^2}+\frac{\rho_0}{\alpha_{11}}\left(\Omega'^2+\omega^2\right)\right)U=0. \qquad (6)$$

The, put

$$U(r)=r^{-m}\Phi(r), \qquad (7)$$

where U(r) is given in terms of m due to in-homogeneity of material.
We have

$$r^2\frac{d^2\Phi}{dr^2}+2r\frac{d\Phi}{dr}+[\lambda^2r^2-n(n+1)]\Phi=0, \qquad (8)$$

where$\lambda^2=\frac{\rho_0}{\alpha_{11}}\left(\Omega'^2+\omega^2\right)$

and

$$n(n+1)=\frac{(\alpha_{22}+\alpha_{33}+2\alpha_{23})-(2m+1)(\alpha_{12}+\alpha_{13})}{\alpha_{11}}+m(m+1).$$

Equation (8) is called spherical Bessel's equation and its general solution is known in the following form:

$$\Phi(r)=A'j_n(\lambda r)+B'y_n(\lambda r), \qquad (9)$$

where$A'$and $B'$are arbitrary constants and$j_n(\lambda r)$and $j_n(\lambda r)$denote spherical Bessel's function of the first and second kind of order n, respectively, which are defined in terms of Bessel's function as follows:

$$j_n(\lambda r)=\sqrt{\frac{\pi}{2\lambda r}}J_{n+\frac{1}{2}}(\lambda r), \ (10)$$

$$y_n(\lambda r)=\sqrt{\frac{\pi}{2\lambda r}}Y_{n+\frac{1}{2}}(\lambda r). \qquad (11)$$

Substituting (11), (10), and (9) into (7), we obtain the complete solution of the differential equation (7) as follows:

$$u(r,t)=r^{-\left(m+\frac{1}{2}\right)}\left[AJ_{n+\frac{1}{2}}(\lambda r)+BY_{n+\frac{1}{2}}(\lambda r)\right]e^{-i\omega t}, \qquad (12)$$

where $A=\sqrt{\frac{\pi}{2\lambda}}A'$ and$B=\sqrt{\frac{\pi}{2\lambda}}B'$.

Substituting (12) into (1), we get the following:

$$\sigma_{rr}=r^{\left(m-\frac{1}{2}\right)}\left[A\left(\lambda\alpha_{11}J_{n-\frac{1}{2}}(\lambda r)+\frac{\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)}{r}J_{n+\frac{1}{2}}(\lambda r)\right)+B\left(\lambda\alpha_{11}Y_{n-\frac{1}{2}}(\lambda r)++\frac{\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)}{r}Y_{n+\frac{1}{2}}(\lambda r)\right)\right]e^{-i\omega t}, \qquad (13)$$

$$\sigma_{\theta\theta}=r^{\left(m-\frac{1}{2}\right)}\left[A\left(\lambda\alpha_{12}J_{n-\frac{1}{2}}(\lambda r)+\frac{\alpha_{22}+\alpha_{23}-\alpha_{12}(m+n+1)}{r}J_{n+\frac{1}{2}}(\lambda r)\right)+B\left(\lambda\alpha_{12}Y_{n-\frac{1}{2}}(\lambda r)+\frac{\alpha_{22}+\alpha_{23}-\alpha_{12}(m+n+1)}{r}Y_{n+\frac{1}{2}}(\lambda r)\right)\right]e^{-i\omega t}, \qquad (18)$$

$$\sigma_{\varphi\varphi} = r^{(m-\frac{1}{2})}\left[A\left(\lambda\alpha_{13}J_{n-\frac{1}{2}}(\lambda r) + \frac{\alpha_{23}+\alpha_{33}-\alpha_{13}(m+n+1)}{r}J_{n+\frac{1}{2}}(\lambda r)\right) + B\left(\lambda\alpha_{13}Y_{n-\frac{1}{2}}(\lambda r) + \frac{\alpha_{23}+\alpha_{33}-\alpha_{13}(m+n+1)}{r}Y_{n+\frac{1}{2}}(\lambda r)\right)\right]e^{-i\omega t}. \quad (14)$$

4. Boundary Conditions and Frequency Equations

We describe the different cases of the boundary conditions and frequency equation for the solid sphere and consider the following transformations:

$$w_1 = \sqrt{w^2\left(\frac{\alpha_{12}}{\alpha_{11}}\right) + \rho_0\frac{\Omega^2}{\alpha_{11}}},$$
$$\omega = \frac{w}{b}\sqrt{\frac{\alpha_{12}}{\rho_0}}\,,\ \Omega' = \frac{\Omega}{b}\ ,\ h = \frac{a}{b}\,,\ \lambda = \frac{w_1}{b}. \quad (15)$$

To make all the quantities dimensionless in (15), we denotewas the dimensionless frequency and derived ω according to [7, 24]. Moreover, we describe the different cases of the boundary conditions and frequency equation for the hollow sphere.

4.1. Free Surface Traction

In this case, a frequency equation is obtained for the boundary conditions, which specify that stresses on the free inner and outer surfaces of the hollow sphere are traction-free, respectively, as follows:

$$\sigma_{rr} = 0 \qquad \text{at} r = a,$$
$$\sigma_{rr} = 0 \qquad \text{at} r = b. \quad (16)$$

Which correspond to free inner and outer surfaces, respectively. From (13), (15), and (16), we obtain the following:

$$A\left[hw_1\alpha_{11}J_{n-\frac{1}{2}}(hw_1) + \left(\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)\right)J_{n+\frac{1}{2}}(hw_1)\right] + B\left[hw_1\alpha_{11}Y_{n-\frac{1}{2}}(hw_1) + \left(\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)\right)Y_{n+\frac{1}{2}}(hw_1)\right] = 0,$$
$$A\left[w_1\alpha_{11}J_{n-\frac{1}{2}}(w_1) + \left(\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)\right)J_{n+\frac{1}{2}}(w_1)\right] + B\left[w_1\alpha_{11}Y_{n-\frac{1}{2}}(w_1) + \left(\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)\right)Y_{n+\frac{1}{2}}(w_1)\right] = 0. \quad (17)$$

Equations (17) also represent a set of two non-homogeneous equations, which give the following frequency equation. The condition for a nontrivial solution of these equations is the determinant coefficient of these integration constants, which must vanish, resulting in the following:

$$\begin{vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{vmatrix} = 0, \quad (18)$$

where

$$D_{11} = hw_1\alpha_{11}J_{n-\frac{1}{2}}(hw_1) + \left(\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)\right)J_{n+\frac{1}{2}}(hw_1),$$
$$D_{12} = hw_1\alpha_{11}Y_{n-\frac{1}{2}}(hw_1) + \left(\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)\right)Y_{n+\frac{1}{2}}(hw_1),$$
$$D_{21} = w_1\alpha_{11}J_{n-\frac{1}{2}}(w_1) + \left(\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)\right)J_{n+\frac{1}{2}}(w_1),$$
$$D_{22} = w_1\alpha_{11}Y_{n-\frac{1}{2}}(w_1) + \left(\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)\right)Y_{n+\frac{1}{2}}(w_1).$$

From (18), we deduce the dimensionless frequency equation as follows:

$$D_{11}D_{22} - D_{12}D_{21} = 0, \qquad (19)$$

which represents an implicit function inw_1. By solving (19), we can obtain the eigenvalues of frequency w.

4.2. Fixed Surface

In this case, a frequency equation is obtained for the boundary conditions specifying that the fixed inner and outer surfaces of the hollow sphere are fixed from displacements, respectively, as follows:

$$\begin{aligned} u(r,t) &= 0 \qquad \text{at} r = a, \\ u(r,t) &= 0 \qquad \text{at} r = b. \end{aligned} \qquad (20)$$

Which correspond to fixed inner and outer surfaces, respectively. From (12) and (20), we obtain the following:

$$\begin{aligned} &AJ_{n+\frac{1}{2}}(\lambda a) + BY_{n+\frac{1}{2}}(\lambda a) = 0, \\ &AJ_{n+\frac{1}{2}}(\lambda b) + BY_{n+\frac{1}{2}}(\lambda b) = 0, \end{aligned} \qquad (21)$$

from which we deduce the dimensionless frequency equation using (15) as follows:

$$\begin{vmatrix} J_{n+\frac{1}{2}}(hw_1) & Y_{n+\frac{1}{2}}(hw_1) \\ J_{n+\frac{1}{2}}(w_1) & Y_{n+\frac{1}{2}}(w_1) \end{vmatrix} = 0.$$

Then,

$$J_{n+\frac{1}{2}}(hw_1)Y_{n+\frac{1}{2}}(w_1) - J_{n+\frac{1}{2}}(w_1)Y_{n+\frac{1}{2}}(hw_1) = 0, \qquad (22)$$

which represents an implicit function in w_1. Equation (20) was solved using *the bisection method* [27].Using numerical methods, we can obtain the eigenvalues of frequency w.

4.3. Inner Fixed Surface and Outer Free Surface

In this case, a frequency equation is obtained for the boundary conditions specifying that the inner fixed surface and outer free surface of the hollow sphere are fixed and free from displacement and stress as follows:

$$\begin{aligned} u(r,t) &= 0 \qquad \text{at} \quad r = a, \\ \sigma_{rr}(r,t) &= 0 \qquad \text{at} \quad r = b. \end{aligned} \qquad (23)$$

From (12), (13), and (23), we obtain the following:

$$\begin{aligned} &AJ_{n+\frac{1}{2}}(\lambda a) + BY_{n+\frac{1}{2}}(\lambda a) = 0, \\ &A\left(\lambda\alpha_{11}J_{n-\frac{1}{2}}(\lambda b) + \frac{\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n)}{b}J_{n+\frac{1}{2}}(\lambda b)\right) + \\ &B\left(\lambda\alpha_{11}Y_{n-\frac{1}{2}}(\lambda b) + \frac{\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n)}{b}Y_{n+\frac{1}{2}}(\lambda b)\right) = 0. \end{aligned} \qquad (24)$$

Then, we deduce the dimensionless frequency equation as follows:

$$\begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix} = 0,$$

where

$$M_{11} = J_{n+\frac{1}{2}}(hw_1),$$

$M_{12} = Y_{n+\frac{1}{2}}(hw_1)$,
$M_{21} = w_1\alpha_{11}J_{n-\frac{1}{2}}(w_1) + [\alpha_{12+}\alpha_{13} - \alpha_{11}(m+n+1)]J_{n+\frac{1}{2}}(w_1)$,
$M_{22} = w_1\alpha_{11}Y_{n-\frac{1}{2}}(w_1) + [\alpha_{12+}\alpha_{13} - \alpha_{11}(m+n+1)]Y_{n+\frac{1}{2}}(w_1)$.
Then,

$$J_{n+\frac{1}{2}}(hw_1)\left[w_1\alpha_{11}Y_{n-\frac{1}{2}}(w_1) + [\alpha_{12+}\alpha_{13} - \alpha_{11}(m+n+1)]Y_{n+\frac{1}{2}}(w_1)\right] +$$
$$-Y_{n+\frac{1}{2}}(hw_1)\left[w_1\alpha_{11}J_{n-\frac{1}{2}}(w_1) + [\alpha_{12+}\alpha_{13} - \alpha_{11}(m+n+1)]J_{n+\frac{1}{2}}(w_1)\right] = 0, \quad (25)$$

which represents an implicit function in w_1. By solving (25), we can obtain the eigenvalues of frequency w.

4.4. Inner Free surface and Outer Fixed Surface

In this case, a frequency equation is obtained for the boundary conditions specifying that the inner free surface and outer fixed surface of the hollow sphere are free and fixed from stress and displacement:

$$\sigma_{rr}(r,t) = 0 \qquad \text{at} \quad r = a,$$
$$u(r,t) = 0 \qquad \text{at} \quad r = b. \quad (26)$$

Based on the above equation, we obtain the following:

$$A\left(\lambda\alpha_{11}J_{n-\frac{1}{2}}(\lambda a) + \frac{\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)}{a}J_{n+\frac{1}{2}}(\lambda a)\right)$$
$$+ B\left(\lambda\alpha_{11}Y_{n-\frac{1}{2}}(\lambda a) + \frac{\alpha_{12}+\alpha_{13}-\alpha_{11}(m+n+1)}{a}Y_{n+\frac{1}{2}}(\lambda a)\right) = 0,$$
$$AJ_{n+\frac{1}{2}}(\lambda b) + BY_{n+\frac{1}{2}}(\lambda b) = 0. \quad (27)$$

Then, we deduce the dimensionless frequency equation using (15) in the followingform :

$$\begin{vmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{vmatrix} = 0,$$

where
$E_{11} = hw_1\alpha_{11}J_{n-\frac{1}{2}}(hw_1) + [\alpha_{12+}\alpha_{13} - \alpha_{11}(m+n+1)]J_{n+\frac{1}{2}}(hw_1)$,
$E_{12} = hw_1\alpha_{11}Y_{n-\frac{1}{2}}(hw_1) + [\alpha_{12+}\alpha_{13} - \alpha_{11}(m+n+1)]Y_{n+\frac{1}{2}}(hw_1)$,
$E_{21} = J_{n+\frac{1}{2}}(w_1)$,
$E_{22} = Y_{n+\frac{1}{2}}(w_1)$.

Then,

$$J_{n+\frac{1}{2}}(w_1)\left[hw_1\alpha_{11}Y_{n-\frac{1}{2}}(hw_1) + [\alpha_{12+}\alpha_{13} - \alpha_{11}(m+n+1)]Y_{n+\frac{1}{2}}(hw_1)\right] -$$
$$-Y_{n+\frac{1}{2}}(w_1)\left[hw_1\alpha_{11}J_{n-\frac{1}{2}}(hw_1) + [\alpha_{12+}\alpha_{13} - \alpha_{11}(m+n+1)]J_{n+\frac{1}{2}}(hw_1)\right] = 0, \quad (28)$$

which represents an implicit function in w_1. By solving (28), we can obtain the eigenvalues of frequency w. A numerical method is used to obtain the eigenvalues of the natural frequency w defined by (19), (22), (25),and (28).

5. The Numerical Procedure

Here, the eigenfrequencies of the problem are obtained by numerically solving the frequency equations. Because these equations are an implicit functional relation of W1 and h, we proceed to find the variation of natural frequency with ratio h. We utilized MATLAB program to analyze W roots of the above equations versus different values of h for the first mode. Moreover, we have adopted the following iterative procedure for numerical computations. For a fixed value of h, we investigated the detrimental equations for various values of the unknown quantity W, starting with the initial value near zero and adding each time a fixed but small increment to that unknown quantity until the signs of determinant value are changed. Then,the bisection method [7,27] is employed to locate the correct root to a chosen number of decimal places. With this root as the initial value, the procedure is repeated to find the next root, and so forth.

Given the geometry and elastic constants of an orthotropic hollow sphere, the frequency equation is an implicit transcendental function for the frequency parameter W. Therefore, for a fixed value of h, the frequency equation for different cases (free-fixed-mixed) is a function of W only. The values of W were 0, 0.5, 1.0, and 1.5. Based on the data for orthotropic material, the results of frequency versus ratio h are plotted in the figures. The elastic constants for an orthotropic material were used as an illustrative example [7].

6. Numerical Results and Discussion

Numerical results have shown the rotation and non-homogeneous effects on frequency w graphically through ratio h of the hollow sphere. The results of the two cases are illustrated in Figures1–8. The basic material properties are as follows:

$$\alpha_{13} = 7.289 \quad GPa, \ \frac{\alpha_{11}}{\alpha_{13}} = 2.34, \quad \frac{\alpha_{12}}{\alpha_{13}} = 0.93, \ \frac{\alpha_{22}}{\alpha_{13}} = 8.18, \quad \rho_0 = 8.93 \quad g\ /cm^3.$$

Using the bisection method, we studied the harmonic vibrations [7, 27]. We recorded the governing equations for future reference. Under the effect of rotation and non-homogeneous, we obtained the frequency equations. The findings revealed that the frequency increased with the increase of h in all cases. The frequency equations have shown that each class of vibration has an infinite number of frequencies. Given the geometry and elastic constants of the sphere, the frequency equations are implicit transcendental equations of frequency parameter w for different boundary conditions (free-fixed-mixed). Figures1–8 represent the variation of frequencies w along ratio h of the inhomogeneous hollow sphere with different values of homogeneity exponent m and rotation **Ω**. Figures 1–8 were plotted for *n=1*. All figures show that the frequencies satisfy the physical phenomena. Figures 1–4 illustrate that when Ω increases, frequencies w decrease. At m = 0.5, the non-dimensional frequency w increases. Moreover, the sphere's rotation greatly affects the non-dimensional frequency.

Figures 5–8 display the effect of rotation Ω on frequency w. They also show that the frequencies increase with the increase of non-homogeneous m. Furthermore, non-dimensional frequencies w increase when ratio h of the sphere increases. The findings prove that the non-homogeneous of the sphere and rotation and ratio h of the shell significantly affect the free vibration frequency of embedded spherical shells.

This study's results were compared with those of previous works given in the absence of rotation and non-homogeneous. The natural frequency results agree with those of Mahmoud [6] and Stavsky and Greenberg [28]. Variations in frequency w with ratio h of non-homogeneous materials have been illustrated graphically and were easily compared with those for the material in the absence of rotation and non-homogeneous [6].These results are of utmost importance to structural design in practical engineering since they illustrate that new material types may withstand demanding conditions, especially in the areas with server restrictions on the structures' dynamics.

7. Conclusion

The free vibration of rotating sphere with non-homogeneous has been studied by using the bisection method; the results and conclusions can be summarized as follows:

a. The harmonic vibrations of the elastic sphere have been studied using a bisection method. The governing equations in spherical coordinates are recorded for future reference. The non-dimensional frequency equations have been obtained under the effects of the rotation. The numerical results of the natural frequency are obtained and discussed in detail in different cases.
b. The rotation plays a significant role in the vibration frequencies. The amplitude of vibration frequencies varies as rotation increase. Presence of rotation restricts the vibration to oscillate near the surface of the cylinder.
c. The results provide a motivation to investigate vibration frequencies of an elastic medium as a new class of application materials. The results presented in this paper should prove useful for researchers in material science, designers of new materials, physicists as well as for those working on the development of elasticity and in practical situations as in the design of microplates and nanoplates and their optimal usage. The used methods in the present article is applicable to a wide range of problems in thermodynamics and elasticity.

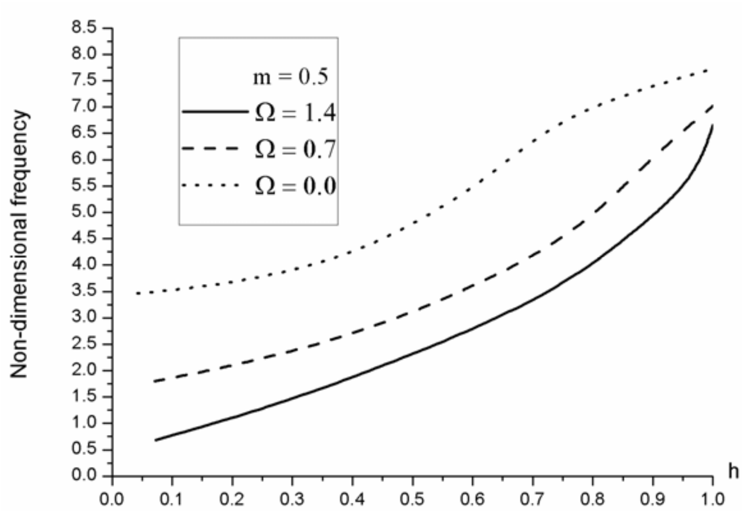

Fig. 1: Variations of non-dimensional frequency **w** with respect to ratio *h* of non-homogeneous material (free traction surfaces), *n = 1*.

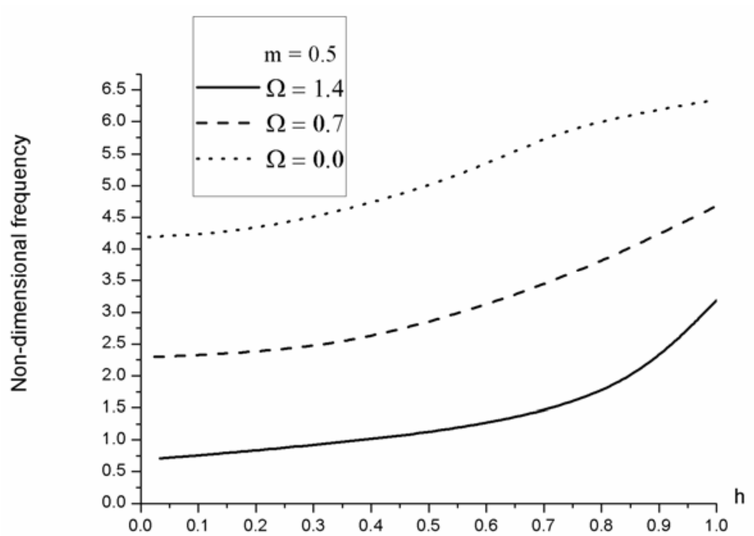

Fig. 2: Variations of non-dimensional frequency **w** with respect to ratio *h* of non-homogeneous material (fixed surfaces), *n = 1*.

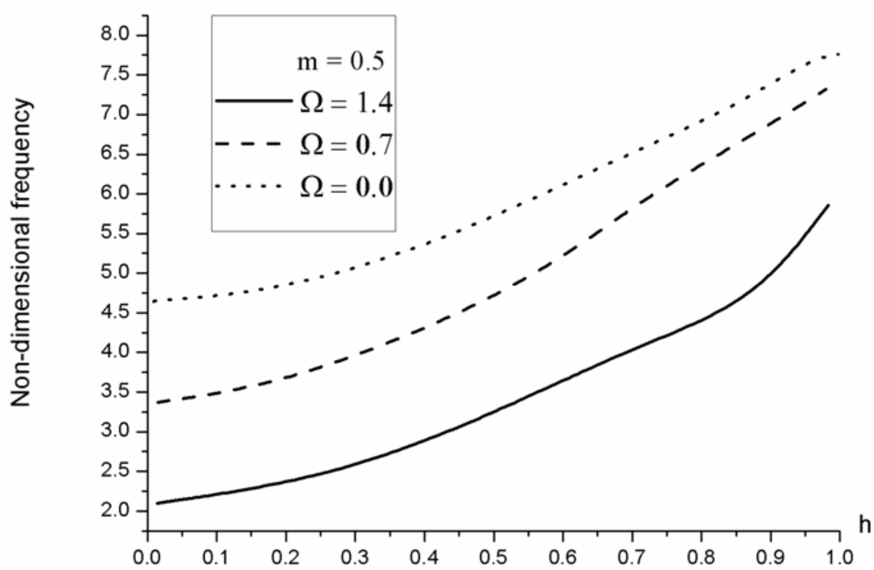

Fig. 3: Variations of non-dimensional frequency $\mathbf{w}$ with respect to ratio *h* of non-homogeneous material (inner fixed surface and outer free surface), *n = 1*.

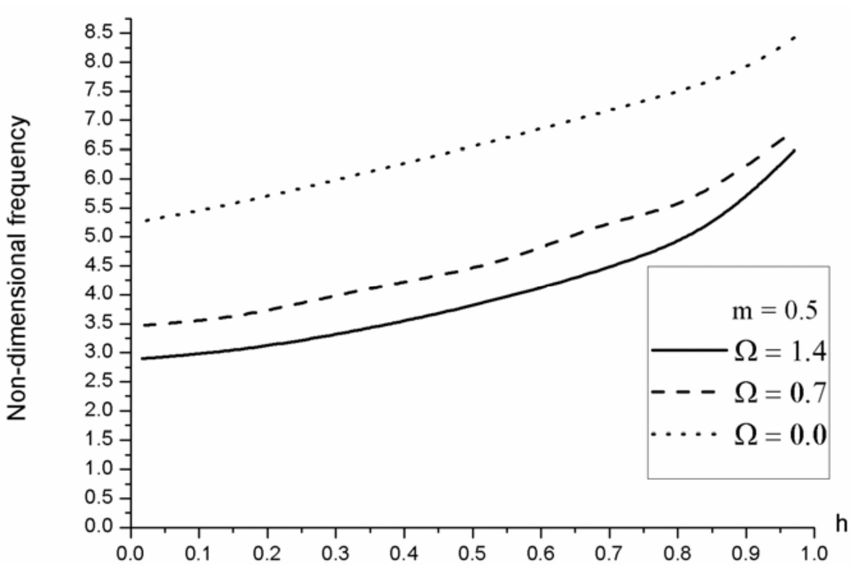

Fig. 4: Variations of non-dimensional frequency $\mathbf{w}$ with respect to ratio *h* of non-homogeneous material (inner free surface and outer fixed surface), *n = 1*.

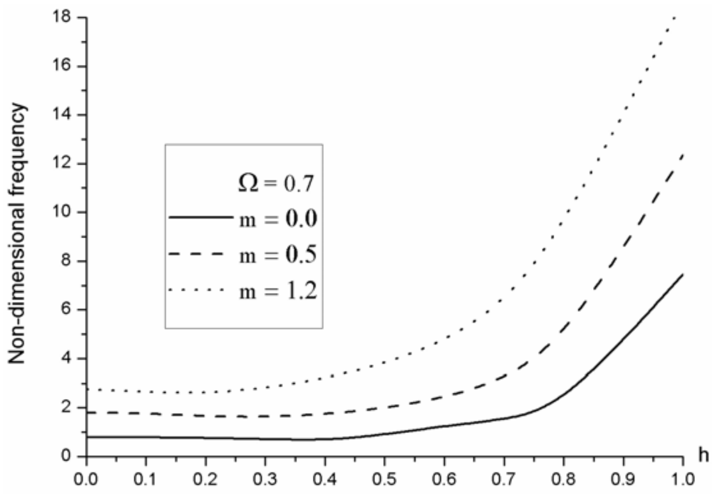

Fig. 5: Variations of non-dimensional frequency $\mathbf{w}$ with respect to ratio *h* of non-homogeneous material (free traction surfaces), *n = 1*.

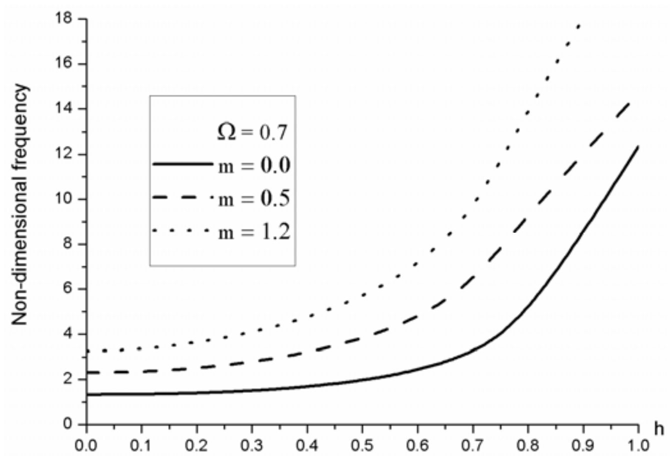

Fig. 6: Variations of non-dimensional frequency $\mathbf{w}$ with respect to ratio *h* of non-homogeneous material (fixed surfaces), *n = 1*.

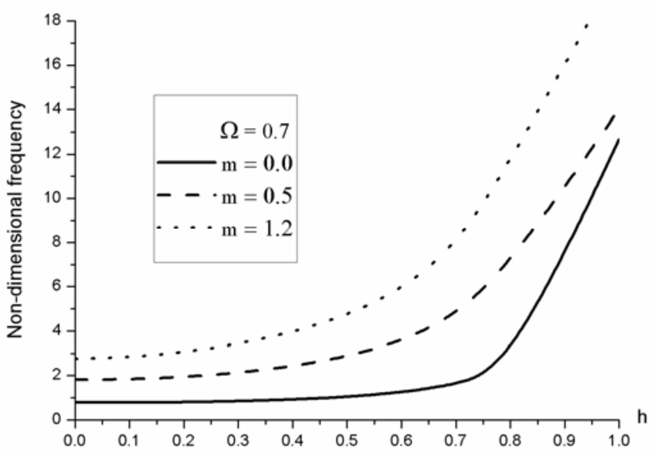

Fig. 7: Variations of non-dimensional frequency **w** with respect to ratio *h* of non-homogeneous material (inner surface fixed and outer surface free), *n = 1*.

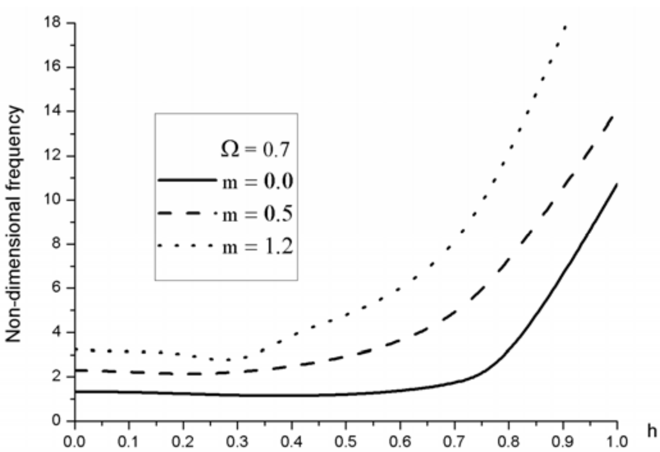

Fig. 8: Variations of non-dimensional frequency **w** with respect to ratio *h* of non-homogeneous material (inner surface free and outer surface fixed), *n = 1*.

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